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EXPERIMENTAL STUDY OF NON-LINEAR VIBRATIONS IN A LOUDSPEAKER CONE

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An experimental study of non-linear vibrations in a loudspeaker cone was made. The sub-harmonic, super sub-harmonic and beat between them are observed. Our experimental results show that non-linear vibrations in a thin shell are related to bending resonance.

1. INTRODUCTION

Considering the revolution shell with monotonically increasing $R_2(s)$, the second principal radius of curvature, there is one turning point in their linear vibration equations for a certain frequency f when it is in the frequency interval (the transition range)

$$\sqrt{E/\rho}[2\pi R_2(b)]^1 \prec f \prec \sqrt{E/\rho}[2\pi R_2(a)]^1 \quad (a \leqslant s \leqslant b), \tag{1}$$

where E is the elastic modulus and ρ is the mass density. Also, for a certain frequency in interval (1) the only turning point position s_* can be found from

$$R_2(s_*) = \sqrt{E/\rho}(2\pi f)^{-1}. \tag{2}$$

We can see that while the frequency f in interval (1) increases, the turning point shifts from the outer edge ($s = b$) of the shell to the inner edge ($s = a$). Ross [1], Zhang and Zhang [2], Tao and Zhang [3] and many others have done much research work on thin shell vibrations in the transition region. They have obtained the turning point solutions for linear vibrations in thin shells. According to the solutions, the bending vibrations occur outside the turning point in the transition region. Furthermore, bending resonance will occur at some frequencies. We have found that the non-linear vibrations of thin shells will first be generated in the neighborhood of bending resonance frequencies.

The loudspeaker cone is a typical revolution thin shell. We discovered the bifurcation and chaos of the sound radiation of loudspeakers in 1986 [4]. However, the signal of sound radiation is actually the superposition of the sound pressures produced by vibrations in all parts of the shell (the paper cone of the loudspeaker). So, the signal of sound radiation does not reflect directly the state of vibrations in every part of the shell. For instance, the

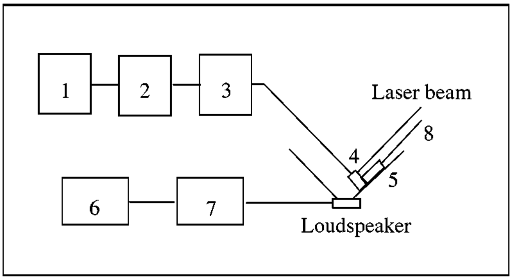


Figure 1. The experimental apparatus: (1) the spectrum analyzer, (2) A/D, (3) the amplifier, (4) the phototube, (5) the modulator of vibration, (6) the generator, (7) the power amplifier, and (8) laser beam.

wavelength of bending vibrations is very short; therefore, the sound pressure produced by bending vibrations will be short circuited, so that it cannot be received at the receiving point. Direct observation of vibrations is necessary for research into non-linear vibrations in thin shells.

In this paper, we introduce an apparatus for directly picking up signals of bending vibrations at any point of the thin shell through the technique of modulated laser. Based on this method we have performed some experiments on non-linear vibrations of thin shells.

2. THE EXPERIMENTAL APPARATUS

The experimental apparatus is shown in Figure 1. The loudspeaker has a curved generatrix. The curvature radius of the generatrix is 170 mm. The inner-edge radius is 7·95 mm and the outer-edge radius is 40·10 mm. The outer edge is free to move. A piece of paper is glued perpendicularly onto the receiving point on the cone surface. The vibrations of the paper modulate the laser beam partly passing through the paper. The modulated beam is received, amplified, and analyzed.

The driving signal voltage of the loudspeaker is normally smaller than 10 V. So the oscillator and the power amplifier are operated in the linear region.

The above-mentioned method of laser-beam modulation is very straightforward. Besides, this method is appropriate for the observation of non-linear vibrations of loudspeakers. Vibrations with large amplitude can be measured with this method because the amplitude being modulated and the vibration displacement of the loudspeaker are on the same order of magnitude.

We chose the observing points, A, B, C and D (Figure 2), so that the shell vibrations both inside and outside the turning point can be observed.

3. THE EXPERIMENTAL RESULTS

The experimental results are shown in Figures 3–10, including the displacement oscillograms and the corresponding frequency spectrograms. In the oscillograms, the amplification factor of display wave height A_m and the scale length of abscissa T_d are given. In the spectrograms, the origin level is L_0 (given in every spectrogram), the origin frequency 0 Hz, the scale length of the ordinate 10 dB and the scale length of the abscissa is 1010 Hz. In these figures, the driving frequency f and driving voltage of the loudspeaker are given.

Some experimental results can be summarized as follows.

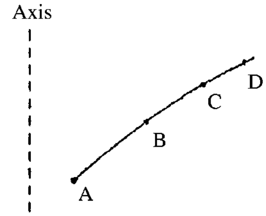


Figure 2. Geometry of loudspeaker shell, A, B, C, D are the positions of observing points.

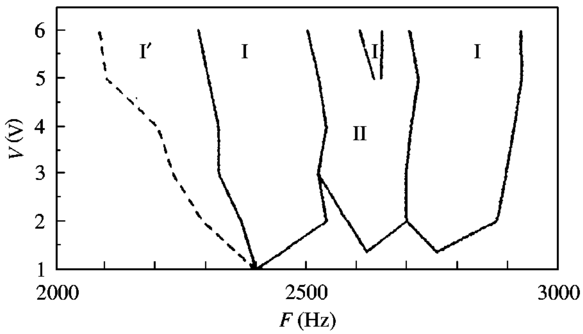
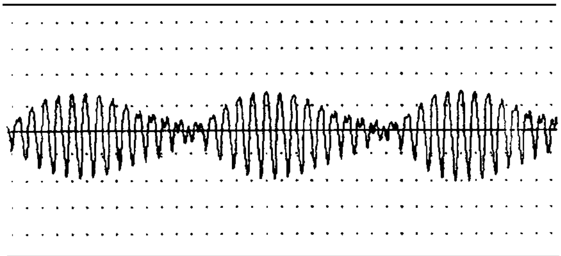


Figure 3. The phase diagram of bifurcation in f–V plane for the loudspeaker: region I is the 1/2 sub-harmonic region, region I′ the hysteresis region of 1/2 sub-harmonic, and region II is the super sub-harmonic region.

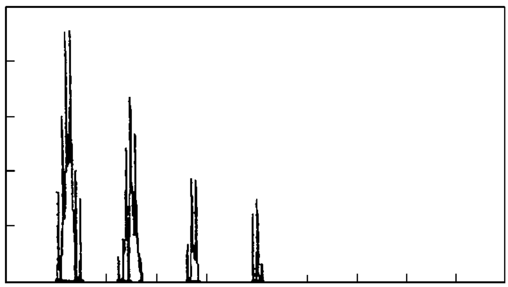
(1) When the driving voltage is equal to 1 V, the loudspeaker is in the linear vibration state. The first, second and third bending resonance frequencies are measured as 2614, 3920, and 5170 Hz respectively (Figure 13). When the driving voltage is increased and the driving frequency approaches the bending resonance frequencies, the sub-harmonics will be generated. Figure 3 is a phase diagram of measured bifurcation. Region I is the 1/2 sub-harmonic region and region I′ is the hysteresis region of 1/2 sub-harmonic. Region II is the super sub-harmonic region. The difference between this phase diagram and the general phase diagrams is that region I is contained in region II under some conditions.

(2) When the driving voltage is 4·5 V and the frequency 2541 Hz, the oscillograms and spectrograms of bending vibrations at points D, C and A are shown in Figures 4–6. The vibration amplitudes at points D and C are larger than that at point A, which means that the energy of non-linear vibrations in the thin shell is concentrated between points D and C. According to our calculation, we can see that point C is very close to the turning point of the cone shell. Obviously, the non-linear vibrations of the cone shell are first generated outside the turning point.

(3) The oscillograms and spectrograms obtained at points A and D are shown in Figures 7 and 8. (The driving voltage is equal to 4·5 V, and the frequency 2810 Hz.) The 1/2 sub-harmonic is 28 dB larger than the fundamental wave at point D. So the oscillogram is a sine wave diagram in which the frequency is equal to 1/2 sub-harmonic (Figure 7). However, at point A the level of 1/2 sub-harmonic is only 5 dB larger than the fundamental wave and it is 22 dB less than that at point D, which further indicates that the non-linear vibrations in thin shells essentially exist outside the turning point of the thin shell.



(a)



(b)

Figure 4. (a) The transverse displacement at point D ($A_m = 1$, $T_d = 0{\cdot}879$ ms/div, $f = 2541$ Hz, 4·5 V). (b) The spectrum of transverse displacement at point D ($L_0 = 20$ dB, $f = 2541$ Hz, 4·5 V).

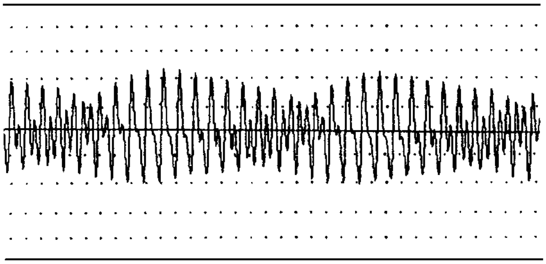


Figure 5. The transverse displacement at point C ($A_m = 2$, $T_d = 0{\cdot}738$ ms/div, $f = 2541$ Hz, 4·5 V).

(4) When the driving frequency approaches 3920 or 5170 Hz, there are more sub-harmonics produced, which are shown in Figures 9 and 10.

(5) When the driving frequency approaches the first bending resonance frequency, two sub-harmonics are generated. The arithmetic sum of their frequencies equals the fundamental frequency and the difference between them is very small, so a “beat” is produced (Figures 4–6).

4. ANALYSIS AND DISCUSSION

Some of the causes and phenomena of the non-linear vibrations of the loudspeaker cone may be explained preliminarily using the theory of linear vibrations of shells.

(1) According to the linear vibration theory of thin shells, below the transition range the bending vibrations are only boundary-layer effects, and in the transition range the

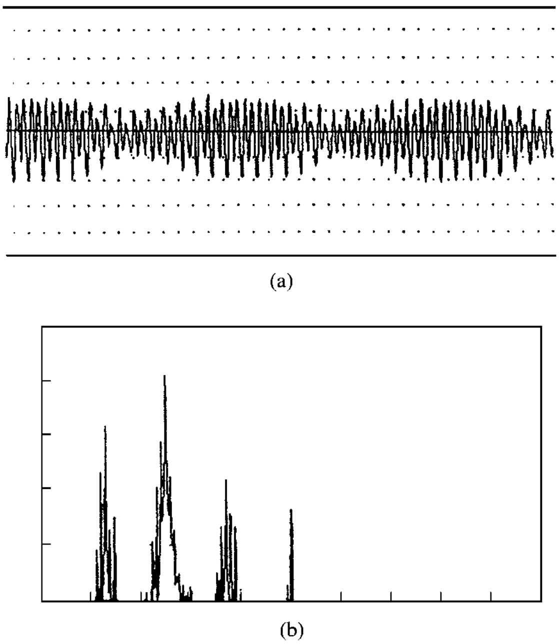


Figure 6. The axial displacement at point A ($A_m = 20$, $T_d = 0{\cdot}790$ ms/div, $f = 2541$ Hz, 4·5 V). (b) The spectrum of axial displacement at point A ($L_0 = 0$ dB, $f = 2541$ Hz, $V = 4{\cdot}5$ V).

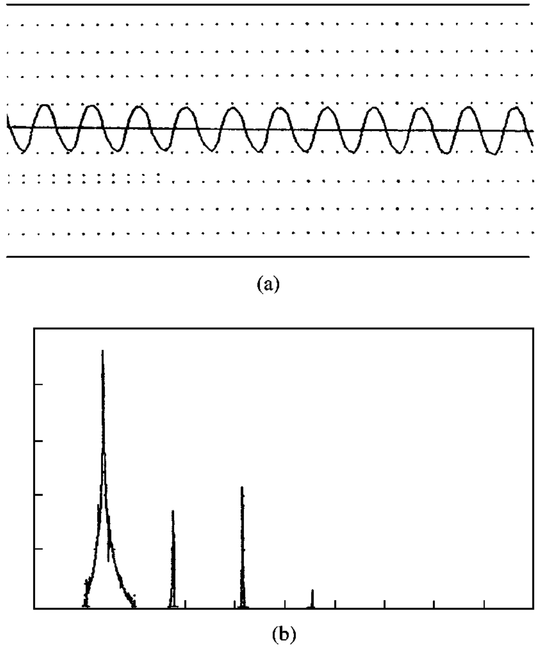


Figure 7. (a) The transverse displacement at point D ($A_m = 1$, $T_d = 0{\cdot}237$ ms/div, $f = 2810$ Hz, 4·5 V). (b) The spectrum of transverse displacement at point D ($L_0 = 20$ dB, $f = 2810$ Hz, 4·5 V).

non-decaying bending wave appears in the interval of $s > s_*$ (as mentioned earlier, s_* is the position of the turning point, which shifts inward as the frequency increases). The frequency response of the strain-energy coefficient η can illustrate this more clearly, η was defined by

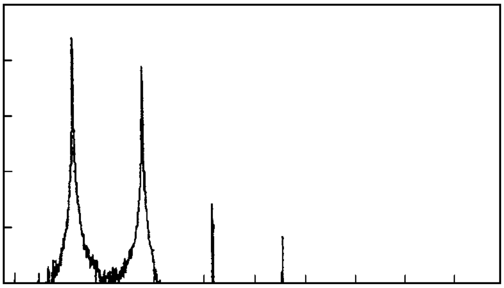


Figure 8. The spectrum of axial displacement at point A ($L_0 = 0$ dB, $f = 2810$ Hz, 4·5 V).

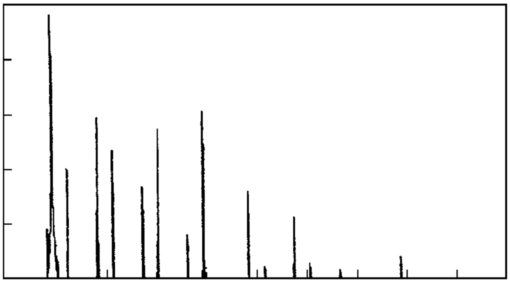


Figure 9. The spectrum of transverse displacement at point D ($L_0 = 25$ dB, $f = 3990$ Hz, 5 V).

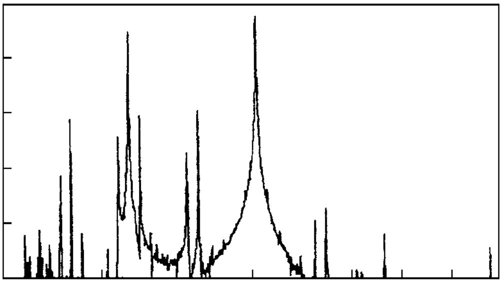


Figure 10. The spectrum of axial displacement at point A ($L_0 = 0$ dB, $f = 5170$ Hz, 8·5 V).

Kalnins [5] as

$$\eta = \frac{V_B}{V_B + V_S}, \tag{3}$$

where V_B is the strain energy due to bending and V_S is the strain energy due to stretching of the middle surface of the cone. The frequency response of the coefficient η of the loudspeaker that we have used is calculated in Figure 11. The interval $[f_{tb}, f_{ta}]$ is the transition range. From Figure 11 we can see that η is only about 0·5% below the transition region and that η increases very fast in the transition region, which means that the bending vibrations are produced in transition region. Therefore, the non-linear bending vibrations will also be produced in the transition region under certain conditions.

(2) The bending solutions for vibrations of thin shells are fast varying and the membrane solutions are slowly varying. It means the bending wavelength is short and the stretching

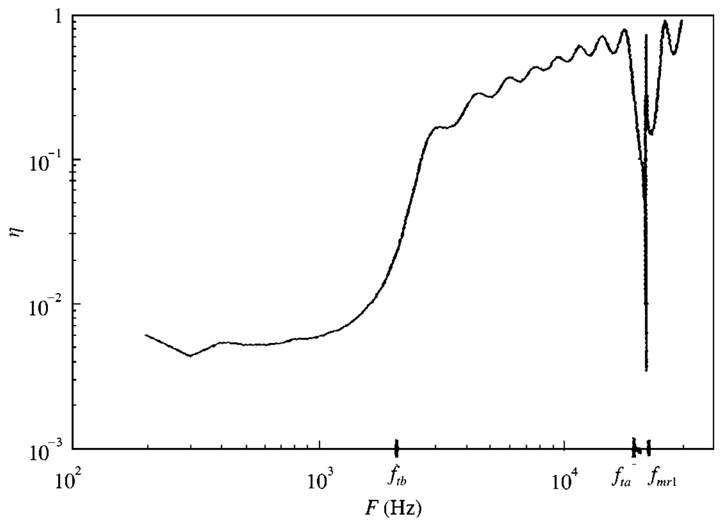


Figure 11. The calculated frequency response of the strain-energy coefficient η.

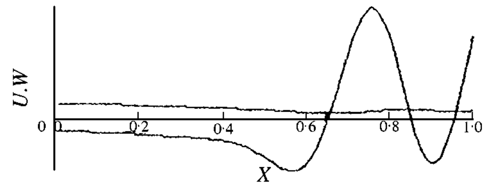


Figure 12. The calculated transverse and longitude displacement patterns of the cone at the third bending resonance.

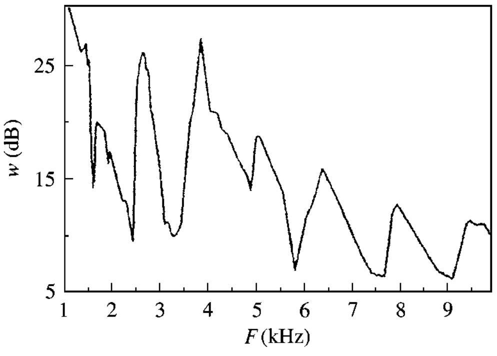


Figure 13. The measured frequency response of the transverse displacement w at point D.

wavelength is long. Furthermore, for the bending solutions the amplitude of transverse displacement w is much larger than that of the longitude displacement u, and for the membrane solutions they are on the same order of magnitude. Figure 12 shows the calculated displacement w and u patterns of the axisymmetric vibrations of the loudspeaker cone. The $*$ in Figure 12 denotes the position of the turning point. Figure 12 also indicates that the bending vibrations exist only outside the turning point, as mentioned in discussion 1. Figure 13 is the measured frequency response of the transverse displacement at point

D when the driving voltage is 1 V, so that the shell vibrations are linear. We can see that the displacement amplitude at the bending resonance frequencies is much larger than that at non-resonance states. Since the value of the angle displacement $\partial w/\partial s$ is large due to both the short bending wavelength and the big amplitude of the transverse displacement at the bending resonance when the driving frequency is close to the bending resonance frequencies, the square term of the angle displacement $\partial w/\partial s$, the main non-linear factor for axisymmetric thin shell vibrations, cannot be neglected in the geometric relation between the strain component ε_1 and the displacement. So the non-linear phenomena are first produced at bending resonance and exist essentially outside the turning point of the cone shell.

ACKNOWLEDGMENT

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